LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600 034

B.Sc.DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2018

16/17/18UST1MC02 / ST 1503- PROBABILITY AND RANDOM VARIABLES

Date: 31-10-2018 Time: 09:00-12:00 Dept. No.

Max.: 100 Marks

SECTION - A (10 x 2 = 20 Marks) Answer ALL questions. All questions carry equal Marks

- 1. What is meant by sample space?
- 2. Give the axioms of probability.
- 3. State the classical definition of probability.
- 4. What is the probability of rolling a 5 when a die is rolled?
- 5. Define conditional probability.
- 6. What do you understand by independent events?
- 7. State the multiplication law of probability.
- 8. A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, NOT replaced and then another pen is drawn, what is the probability of drawing 2 blue pens and 1 black pen?
- 9. Define discrete random variables.
- 10. Give the concept of mathematical expectation.

SECTION –B (5 x 8 = 40 Marks) Answer any FIVE questions

- 11. State and Prove addition law of probability. (8 Marks)
- 12. (i) Prove that $P(A) = P(A \cap B) + P(A \cap \overline{B})$ (4 Marks)

(ii) Prove that $P(B) = P(A \cap B) + P(B \cap \overline{A})$ (4 Marks)

13. (i) What is the probability of the occurrence of a number that is odd or less than 5 when a fair die is rolled? (4 Marks)

(ii) We have numbered cards from 1 to 20 and one is picked at random. Find the probability that the

card picked is numbered a multiple of 2 or 5. (4 Marks)

14. Explain conditional probabilities with example.	(8 Marks)
15. Explain what is meant by independence of events?	(8 Marks)
16. State and prove Baye's Theorem of probability.	(8 Marks)
17. State and prove addition theorem of Mathematical expectation.	(8 Marks)
18. State and prove Chebyshev's inequality.	(8 Marks)

SECTION - C (2 x 20 = 40 Marks) Answer any TWO questions

19. (a) There are 3 blue, 1 white and 4 red identical balls inside a bag. If two balls are taken out of the bag consecutively, what is the probability to have 1 blue and 1 white ball?

(5 Marks)

(b) Three groups of children contain respectively

- (i) 3 girls and 1 boy
- (ii) 2 girls and 2 boys
- (ii) 1 girl and 3 boys.

One child is selected at random from each group. Find the probability of getting 1 girl and 2 boys among the selected.

(10 Marks)

(c) There are 5 blue, 5 green and 5 red books on a shelf. Two books are selected randomly. What is the probability of choosing two books of different colors?

(5 Marks)

20. (a) (i) What are the properties of classical probability of an event A? (5 Marks)

(ii) If A and B are independent events, establish the independence of their complements $\bar{A}and\bar{B}$?

(5 Marks)

(b) Let two honest coins, marked 1 and 2, be tossed together. The four possible outcomes are T_1T_2 , T_1H_2 , H_1T_2 , H_1H_2 . (T_1 indicates toss of coin 1 resulting in tails; similarly T_2 etc.) We shall treat that all these outcomes are equally likely; that is the probability of occurrence of any of these four outcomes is ¹/₄. (Treating each of these outcomes as an event, we find that these events are mutually exclusive and exhaustive). Let the event *A* be 'not H_1H_2 ' and *B* be the event 'match'. (Match comprises the two outcomes T_1T_2 , H_1H_2). Find *P* (*B*/*A*). Are *A* and *B* independent?

(10 Marks)

21. (a) A jar of marbles contains 4 blue marbles, 5 red marbles, 1 green marble, and 2 black marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. Find the probability for the following:

(i) P(green and red) (ii) P(blue and black)

(b) The urns contain 6 green, 4 black; 4 green, 6 black and 5 green, 5 black balls respectively.Randomly selected an urn and a ball is drawn from it.

i) What is the probability it is green?

ii) Given that the ball is drawn is green, find the probability it is drawn from urn II.

22. (a) Suppose that a pair of fair dice is to be tossed, and let the random variable *X* denote the sum of the points. Obtain the probability distribution for *X*.

(b) The distribution function for a random variable X is

$$F(x) = \begin{cases} 1 - e^{-2x}, x \ge 0\\ 0, \qquad x < 0 \end{cases}$$

Find (i) the density function, (ii) the probability that X > 2, and (iii) the probability that $-3 \le X \le 4$.

(10 Marks)

(5 Marks)

(c) In a lottery there are 200 prizes of \$5, 20 prizes of \$25, and 5 prizes of \$100. Assuming that 10,000 tickets are to be issued and sold, what should be the minimum price to be fixed for ticket on a no – gain, no – loss basis.

(5 Marks)

(10 Marks)

(10 Marks)